

Killer Geometries in Competing Species Dynamics

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Abstract

We discuss a cellular automata model to study the competition between an emergent better fitted species against an existing majority species.. The model implement local fights among small group of individual and a synchronous random walk on a 2D lattice. The faith of the system, i.e. the spreading or disappearance of the species is determined by their initial density and fight frequency. The initial

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density of the emergent species has to be higher than a critical threshold for total spreading but this value depends in a non-trivial way of the fight frequency. Below the threshold any better adapted species disappears showing that a qualitative advantage is not enough for a minority to win. No strategy is involved but spatial organization turns out to be crucial. For instance at minority densities of zero measure some very rare local geometries which occur by chance are found to be killer geometries. Once set they lead with high probability to the total destruction of the preexisting majority species. The occurrence rate of these killer geometries is function of the system size. This model may apply to a large spectrum of competing groups like smoker-non smoker, opinion forming, diffusion of innovation setting of industrial standards, species evolution, epidemic spreading and cancer growth.

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1 Introduction

The social behavior of a group of persons is certainly related in part to the fact that each individual has its own autonomy and perception of the environment. However the global behavior may also reflect some “mechanical” response of each individual to the specific situation it is confronted with. Such a collective behavior may possibly be captured at by some cellular automaton model (see [1] for a general introduction to cellular automata) provided the rules, to which each individual obeys, are suitably chosen.

Here we address the generic problem of the competing fight between two different groups over a fixed area. We present a “voter model” which describes the dynamical behavior of a population with bimodal conflicting interests and study the conditions of extinction of one of the initial groups [2, 3]. Note that other interesting applications can be addressed with this model, such as the problem of cancer tumor growth [4].

2 The Model

Our model can be illustrated by the smoker - non smoker confrontation. In a small group of persons a majority of smokers will usually convince the few others to allow them to smoke making smoking the non smokers at least

passively and vice versa. But each time an equal number of smokers and non-smokers meet an uncertainty occurs. In that case it may be assumed that a social trend will decide between the two attitudes. In the US, smoking is viewed as a overall disadvantage whereas in France it is rather well accepted. In other words, there is a bias that will select the winner attitude in an even situation. In our example, whether one studies the French or US case, the bias is in favor of the smokers or the non-smokers, respectively. In our model such a smoking non-smoking choices is considered for each new social encounter.

The same mechanism can be associated with the problem of two competing standards. The choice of one or the other standard is often driven by the opinion of the majority of people one meets. But when the two competing systems are equally represented, the intrinsic quality of the product becomes decisive. In that case, price and technological advantage play the role of a bias.

Here we consider the simpler case of four-person encounters in a spatially extended system in which the actors (species A or B) move randomly. Initially, the B species is present with density b_0 and the A species with density $1 - b_0$. The B individuals are supposed to have a qualitative advantage over the A s but are less numerous. The question we want to address is what is the minimal density b_0 which make the B s win over A (i.e. invade the entire system at the expense of A individuals). Thus this model represents a process of spatial contamination of opinion. Continuous approach have been also considered [11]

The CA rule we propose here [2] to describe this process is derived from a model by Galam [6], in which the four individuals involved in a tournament are randomly chosen among the current population, whose composition in A or B type of person evolves after each confrontation. The density threshold for an invading emergence of B is $b_c = 0.23$ if the B group has a qualitative bias over A . However, with a spatial distribution of the species, even if $b_0 < b_c$, B can still win over A provided that it strives for confrontation. However, when the qualitative advantage is not enough to win, a geographic as well as a definite degree of aggressiveness is instrumental to overcome the less fitted majority.

The model we use to describe the two populations A and B influencing each other or competing for some unique resources, is based on the diffusion automaton proposed in [1, 2]. Particles have two possible internal states (± 1), coding for the A or B species, respectively. Individuals move on a two-

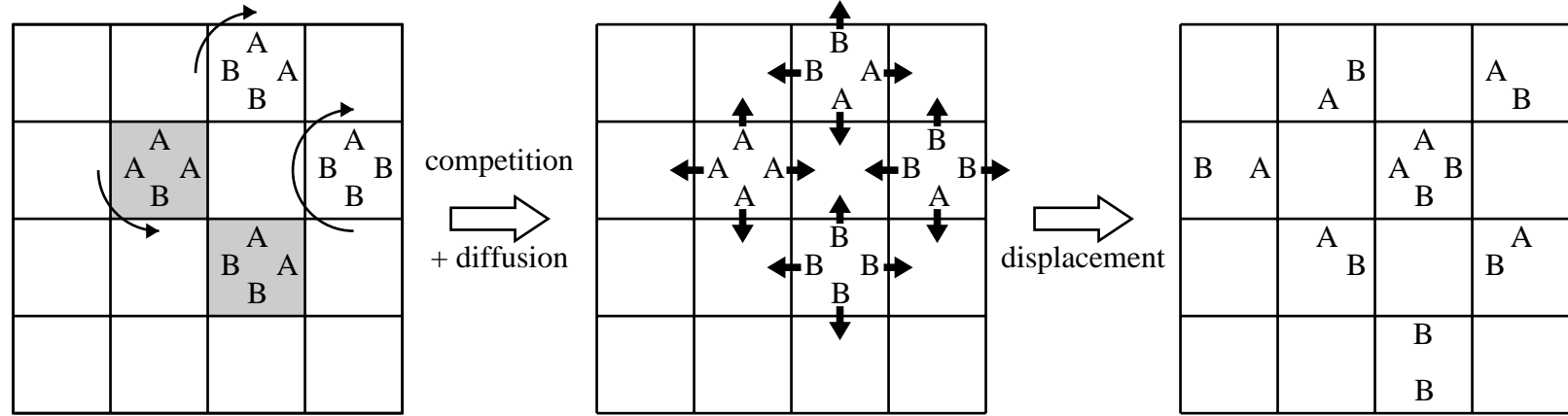


Figure 1: Sketch of the model rule. The symbols A and B denote the two types of individuals. A confrontation take place in all gray cells and results in a local victory of one species. Then, in all cells a random re-direction of the individuals is performed (with a rotation of the configuration by 0, 90, -90 or 180 degrees), followed by a jump to the nearest neighbor cell.

dimensional square lattice. At each site, there are always four individuals (any combination of A 's and B 's is possible). These four individuals all travels in a different lattice direction (north, east, south and west). The situation is illustrated in Fig. 1

The interaction takes place in the form of “fights” between the four individuals meeting on the same site. At each fight, the group nature (A or B) is updated according to the majority rule when possible, otherwise with a bias in favor of the best fitted group. The rules are,

- The local majority species (if any) wins:

$$nA + mB \rightarrow \begin{cases} (n+m)A & \text{if } n > m \\ (n+m)B & \text{if } n < m \end{cases}$$

where n and m are integers satisfying the constraint $n + m = 4$.

- When there is an equal number of A and B on a site, B wins the confrontation with probability $1/2 + \beta/2$. The quantity $\beta \in [0, 1]$ is the bias accounting for some advantage (or extra fitness) of species B .

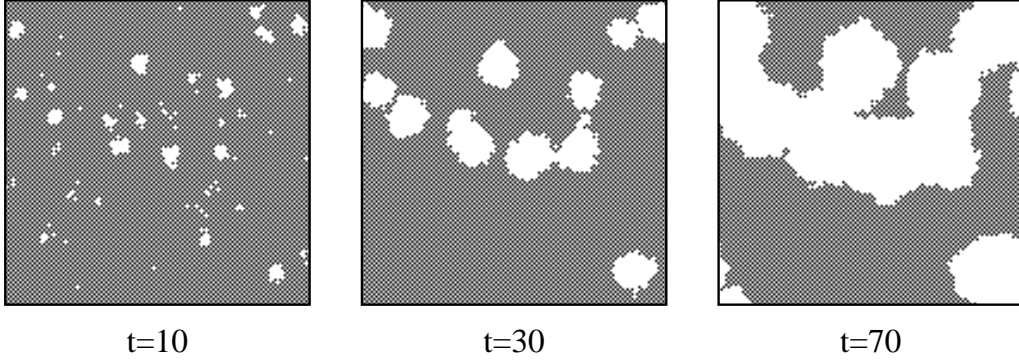


Figure 2: Configurations of the CA model, at three different times. The A and B species are represented by the gray and white regions, respectively. The parameters of the simulation are $b_0 = 0.1$, $k = 0.5$ and $\beta = 1$.

Above rule is applied with probability k . Thus, with probability $1 - k$ the group composition does not change because no fight occurs. Between fights both population agents diffuse on the lattice, by randomly choosing a new direction (see [1] for more detail).

The behavior of this model is illustrated in Fig. 2. The current configuration is shown at three different time steps. We can observe the growth of dense clusters of B invading the system.

It is clear that the model richness comes from the even confrontations. If only odd fights would happen, the initial majority population would always win after some short time. The key parameters of this model are (i) k , the aggressiveness (probability of confrontation), (ii) β , the B 's bias of winning a tie and (iii) b_0 , the initial density of B .

The strategy according to which a minority of B 's (with yet a technical, genetic, persuasive advantage) can win against a large population of A 's is not obvious. Should they fight very often, try to spread or accept a peace agreement? We study the parameter space by running the cellular automaton.

In the limit of low aggressiveness ($k \rightarrow 0$), the particles move a long time before fighting. Due to the diffusive motion, correlation between successive fights are destroyed and, for $\beta = 1$, B wins provided that $b_0 > 0.23$. This is the mean-field level of our dynamical model which corresponds to the theoretical calculations made in [6].

More generally, we observe that B can win even when $b_0 < 0.23$, provided it acts aggressively, i.e. by having a large enough k . Thus, there is a critical density $b_{death}(k) < 0.23$ such that, when $b_0 > b_{death}(k)$, all A are eliminated in the final outcome. Below b_{death} , B loses unless some specific spatial configurations of B 's are present.

Therefore the growth of species B at the expense of A is obtained by a spatial organization. Small clusters that may accidentally form act as nucleus from which the B 's can develop. In other words, above the mean-field threshold $b_c = 0.23$ there is no need to organize in order to win but, below this value only condensed regions will be able to grow. When k is too small, such an organization is not possible (it is destroyed by diffusion) and the strength advantage of B does not lead to success.

Figure 3 summarizes, as a function of b_0 and k , the regions where either A or B succeeds. It is found by inspection that the separation curve satisfies the equation $(k + 1)^7(b_0 - 0.077) = 0.153$.

It is also interesting to study the time needed to annihilate completely the looser. Here, time is measured as the number of fights per site (i.e. kt where t is the iteration time of the automaton). We observe that the dynamics is quite fast and a few units of time are sufficient to yield a collective change of opinion.

Following the same methodology, more complicated interactions between individuals can be investigated. The case of a non-constant bias is quite interesting and is described in [2] and illustrated in Fig. 3 (left). In this latter case, the bias decreases locally if in a neighborhood of diameter ℓ there are enough B . There is a re-entrance phenomena which shows that aggressivity, if too large, can then be detrimental to the B species.

3 Finite size effects

In this section we demonstrate the essential role played by finite size systems in the context of the present model and we show that our model can be described in terms of a probabilistic phase diagram which reduces to a trivial situation when the system size goes to infinity. A more detailed analysis is given in [3].

A possible conclusion is that some socio-economical systems may be characterized by a strong sensitivity to system size. For instance, the macroscopic behavior may change dramatically whether the system is just large or if it is

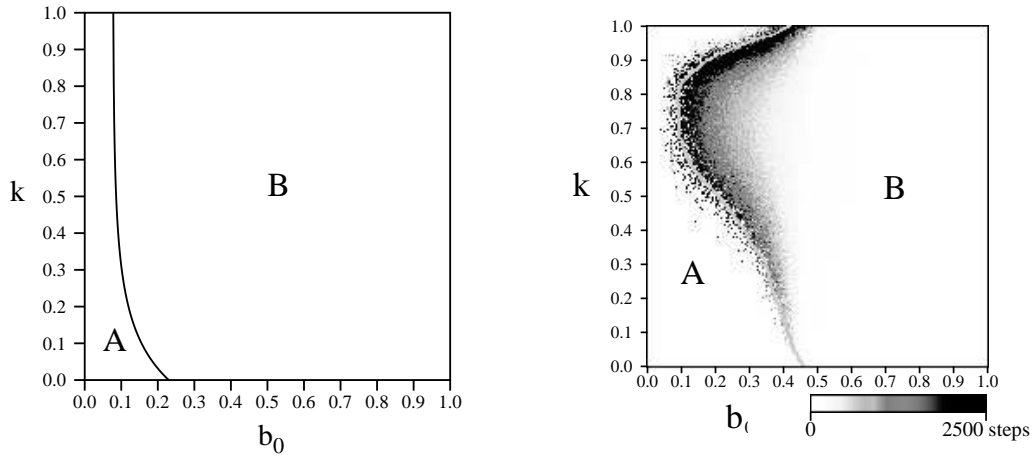


Figure 3: left: Phase diagram for the model, with $\beta = 1$. The curve delineates the regions where, on the left, A wins with high probability and, on the right, B wins with probability one. The outcome depends on b_0 , the initial density of B and k , the probability of a confrontation. Right: Same as the left panel but for a bias computed according to the B density on a local neighborhood of size $\ell = 7$. The gray levels indicate the time to eliminate the defeated species (dark for long time).

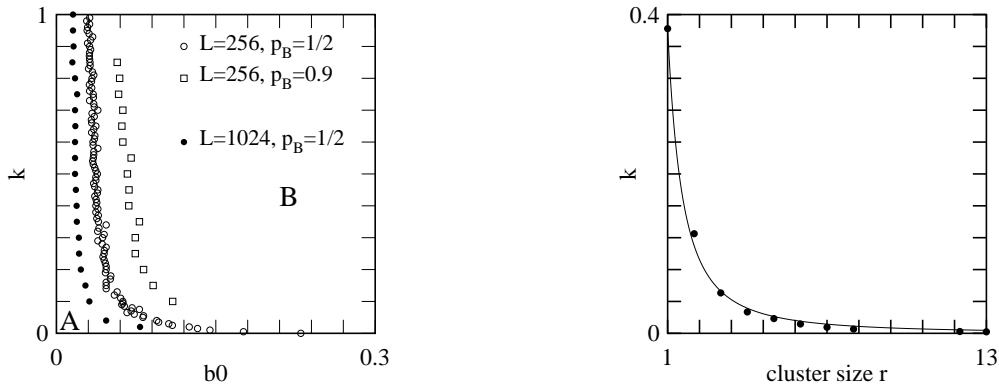


Figure 4: Left: Probabilistic stationary state phase diagram for a systems of size $L = 256$ and $L = 1024$. Contour lines for $p_B = 0.5$ and/or 0.9 are shown. The region marked B indicates that p_B is large whereas it is small in region A . Right: Critical size r of a single B cluster that invades the system with probability 0.9 , as a function of the aggressiveness k . Dots are the results of the CA model and the solid line is an empirical fit: $k = 1/(r^{1.8})$.

almost infinite.

The reason of this peculiar property is the existence, in such systems, of statistically very rare configurations which drive the evolution in a new and atypical way. The observation that rare events can develop and reach a macroscopic size has already been noticed in other contexts. Examples are given by the generalized prisoner dilemma problems [7, 8, 9] or the recent work by Solomon [10]. Percolation problems [11] give another example where a qualitative change of behavior is observed in the limit of an infinite system [12].

To illustrate this behavior, we consider a 1D system in which the effect is more pronounced. The rule of the dynamics is a straightforward variation of the above 2D case. We still consider four individuals per cell and in order to conform to the topology restriction we change the motion rule as follows: two individual randomly chosen among the four travel to the left while the two others travel to the right.

Here we study systems of linear size L with periodic boundary conditions. For given values of b_0 and k the dynamics is iterated until a stationary state (either all A or all B) is reached. The interesting point is that the outcome of this experiment is found to be probabilistic: the final state is all B with

probability p_B and all A with probability $1 - p_B$. Also, the value of p_B depends crucially on the system size L . As we shall see, when $L \rightarrow \infty$, p_B is one for the all (b_0, k) plane.

For this reason, a standard phase diagram cannot describe the situation properly. Thus, we propose a description in terms of what we call a probabilistic phase diagram: each point of the (b_0, k) plane is assigned a probability p_B that the final state is entirely B . Ideally, this diagram should be represented as a 3D plot. Instead, in Fig. 4 (left), we show contour lines corresponding to given probabilities p_B . Note that for the same value of p_B , the isoline is shifted to the left as the system size increases.

These data show that if the aggressiveness k is large enough, initial configurations with a quite low density of B 's are able to overcome the large initial majority of the A species. The reason being the presence of B actors which are organized in small clusters such that the diffusion is not effective enough to destroy them. They expand at a rate which makes them win systematically in the fights against A actors. Fig. 4 (right) is obtained by considering a unique initial B cluster of size r in a sea of A 's. The plot shows, for each value of k the critical value of r which ensures that the B cluster will invade all the system with probability 0.9.

The result of Fig. 4 (right) is independent of the system size L and the question is then how often such clusters appear by chance. In a finite size system, with a given random concentration b_0 of B actors, there is always a finite probability for such small clusters to exist in the initial configuration. When this is the case the system will reach a pure B stationary state. The larger the L the more likely it is to observe such a devastating cluster.

The way the separation line in Fig. 4 (left) depends on L has been investigated in Fig. 5 (left). The plot shows the location of the transition line as a function of L for a fixed probability $p_B = 1/2$ and different values of k .

One sees that when L increases, the probabilistic line corresponding to a given probability p_B moves to the left and an extrapolation to an infinite size system leads to a collapse of the transition line with the vertical axis for all values $k \neq 0$. For $k = 0$, one recovers the mean-field transition point $b_{0c} = 0.23$, for all values of $p_B > 0$. This is shown in Fig. 4 for the case $L = 256$, $p_B = 1/2$, and can be confirmed by direct numerical simulations at $k = 0$ (complete mixing of the individual at each time step).

These results show that the respective behaviors of finite size and infinite size systems are qualitatively different. Fig. 5 (right) shows, for a fixed system size $L = 256$, how the critical density b_0 varies with p_B . For two

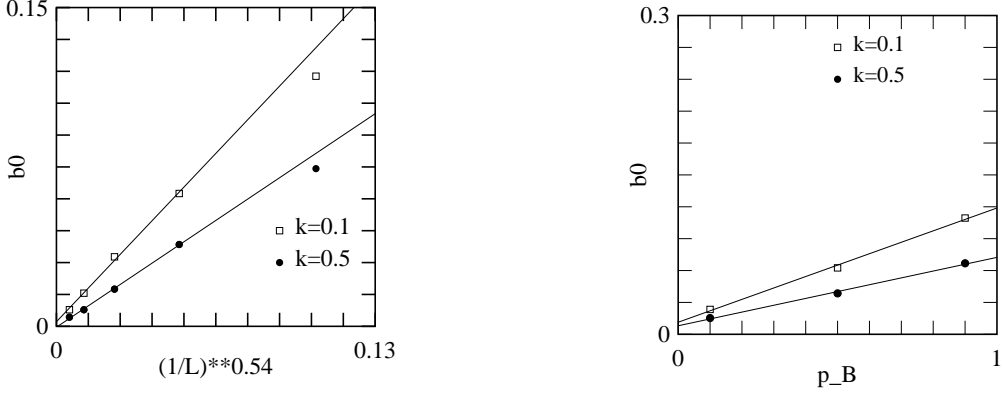


Figure 5: Left: Dependence of the critical density b_0 of B particle as a function of the system size L , for a winning probability $p_B = 0.5$ and two values of k . We see that the $A - B$ separation line moves as $1/(L^{0.54})$. Right: Critical initial density b_0 as a function of B 's probability to win, p_B , for two values of k and $L = 256$. From the assumption of a linear dependence, the value of b_0 for $p_B = 1$ can be interpolated.

values of k , the plot suggests an almost linear dependence.

In [3] we discuss in more detail the question of the appearance of the devastating B clusters, that is the probability $P_L^{(r)}$ to find at least one cluster formed of r consecutive B particles in a system of size L providing that the sites are randomly filled respectively with B particles with probability b_0 and with A particles with probability $a_0 = 1 - b_0$. The result is that $P_L^{(r)} \rightarrow 1$ as $L \rightarrow \infty$, as long as $a_0 \neq 0$.

4 Conclusion

In conclusion, although this model is very simple, it abstracts the complicated behavior of real life agents by capturing some essential ingredients. For this reason, the results we have presented may shed light on the generic mechanisms observed in a social system of opinion making. In particular we see that the correlations existing between successive fights may strongly affect the global behavior of the system and that an organization is the key feature to obtain a definite advantage over the other population. This observation is important. For instance, during a campaign against smoking or an attempt

to impose a new system, it is much more efficient (and cheaper) to target the effort on small nuclei of persons rather than sending the information in an uncorrelated manner.

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